Root-Finding

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Case Study

- Vibrations of a Satellite Boom
- 4th order ordinary differential equation
- Apply boundary conditions
- Solve for roots and then for frequencies

The Approach

$$y(x) = A\cosh(\beta x) + B\sinh(\beta x) + C\cos(\beta x) + D\sin(\beta x)$$

$$y(0) = \frac{dy}{dx}(0) = \frac{d^2y}{dx^2}(L) = \frac{d^3y}{dx^3}(L) = 0$$

- Apply boundary conditions
- Eliminate A, B, C, D
- Obtain equation for β in terms of L
- Solve for β, then for frequencies

Resulting Equation for Frequencies

$$\frac{\cosh(\beta L)+\cos(\beta L)}{\sinh(\beta L)-\sin(\beta L)}=\frac{\sinh(\beta L)+\sin(\beta L)}{\cosh(\beta L)+\cos(\beta L)}$$

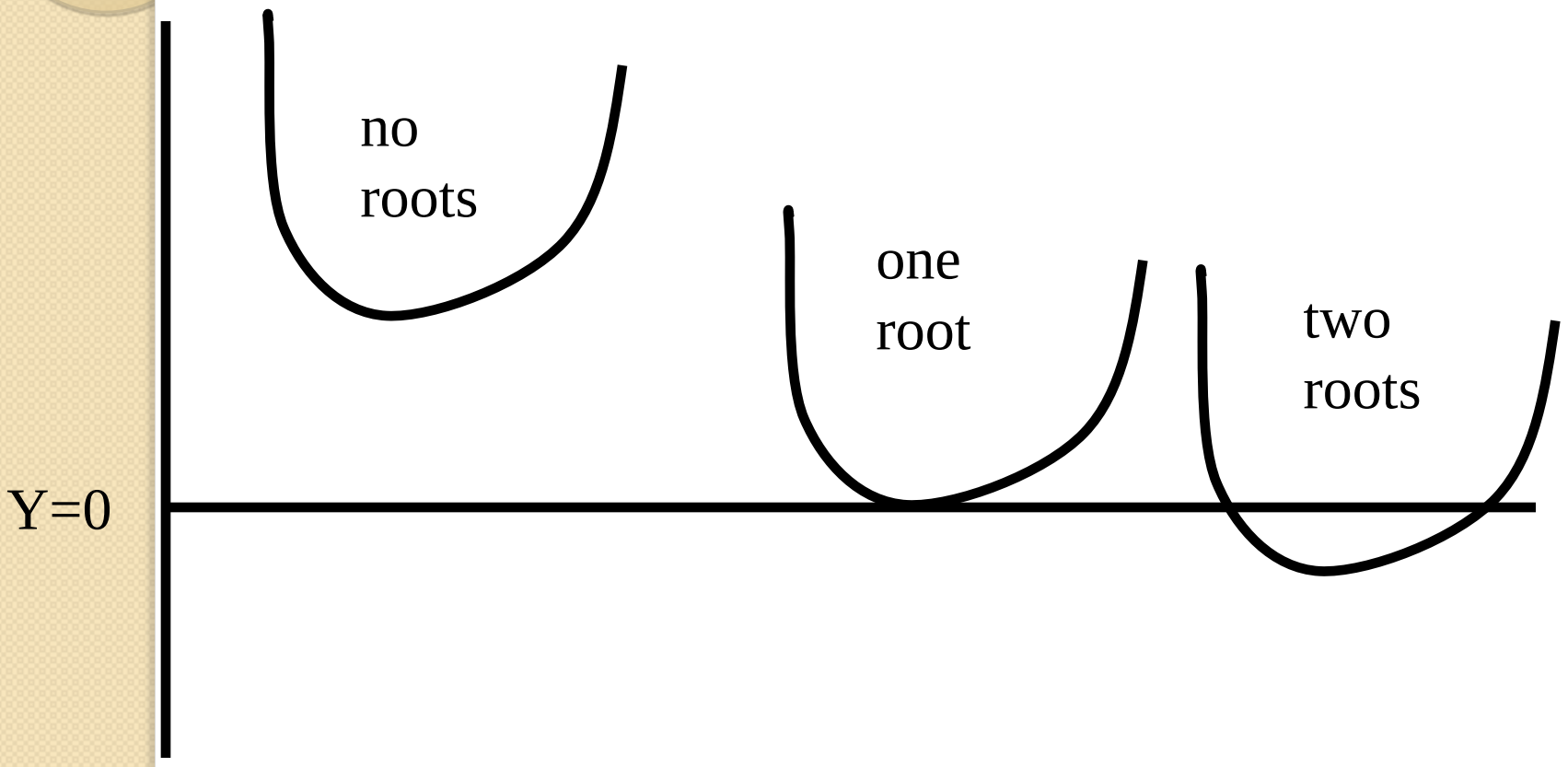
or

$$1+\cosh(\beta L)\cos(\beta L)=0$$

Root Finding

- Goal is to find x such that f(x)=0
- These values of x are called roots or zeroes
- There may be none, 1, more than 1, or an infinite number of roots
- Always plot function first

Some Possible scenarios

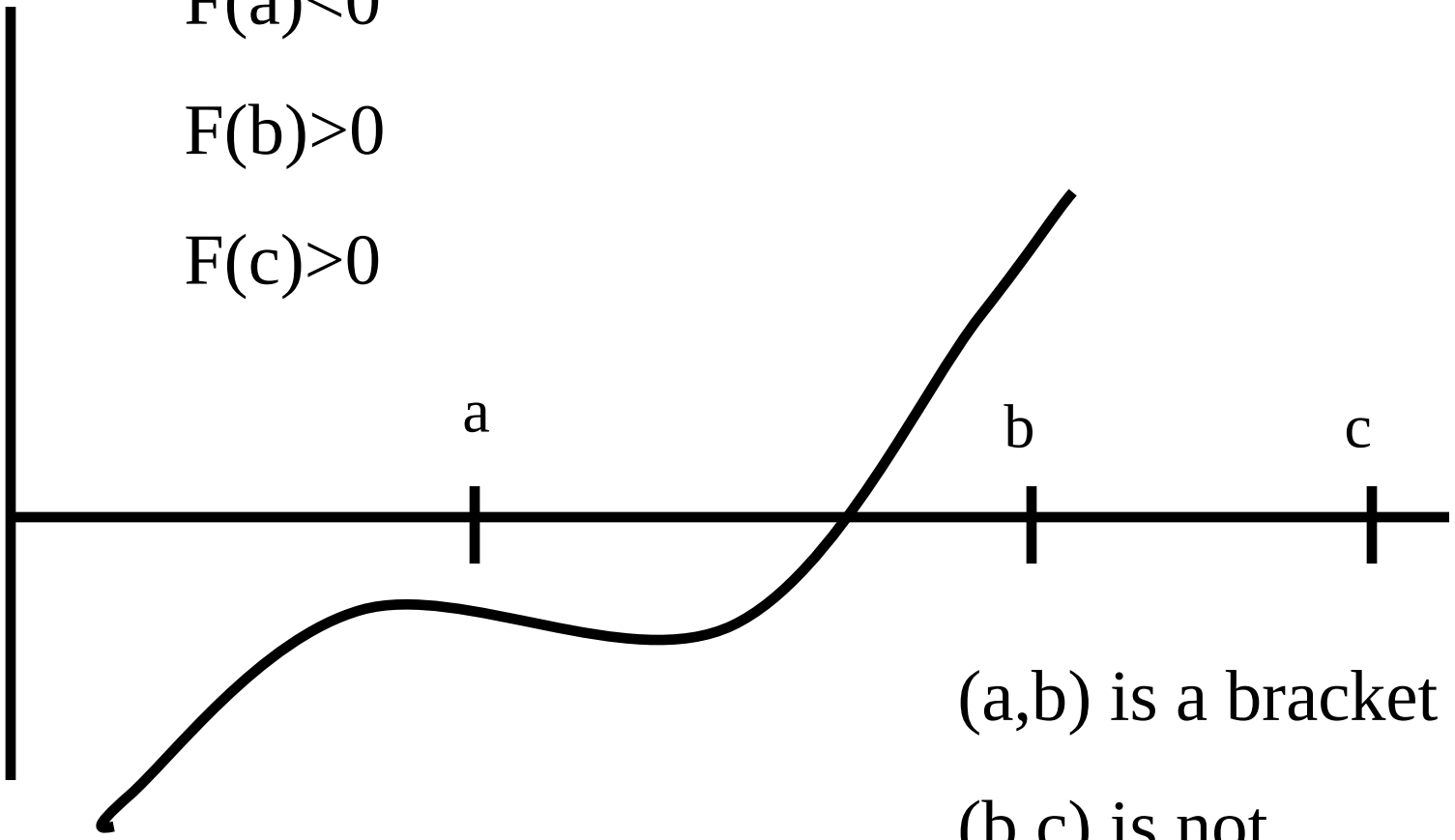


Bracketing

F(a)<0

F(b)>0

F(c)>0

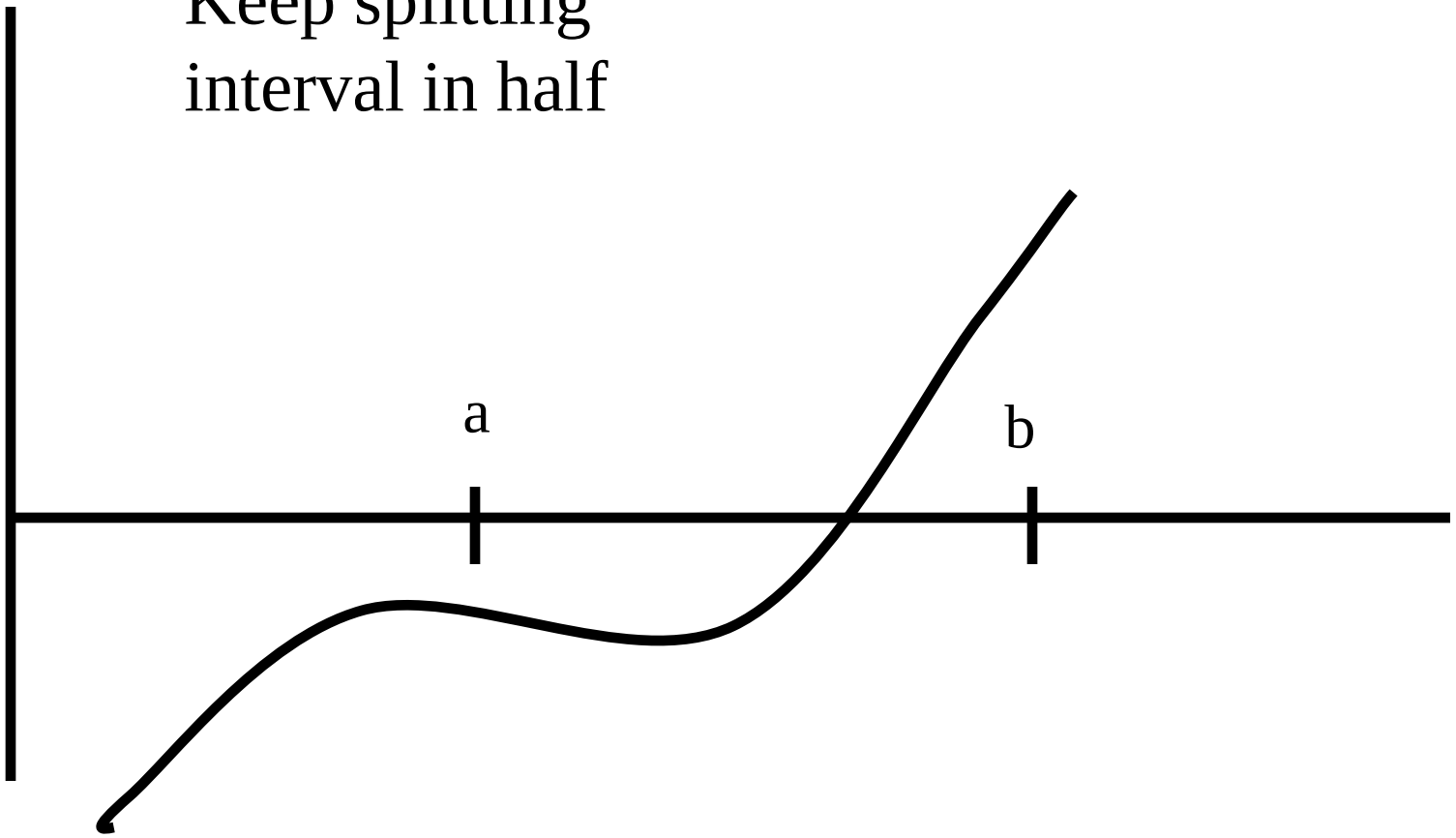


(a,b) is a bracket

(b,c) is not

Bisection

Keep splitting
interval in half



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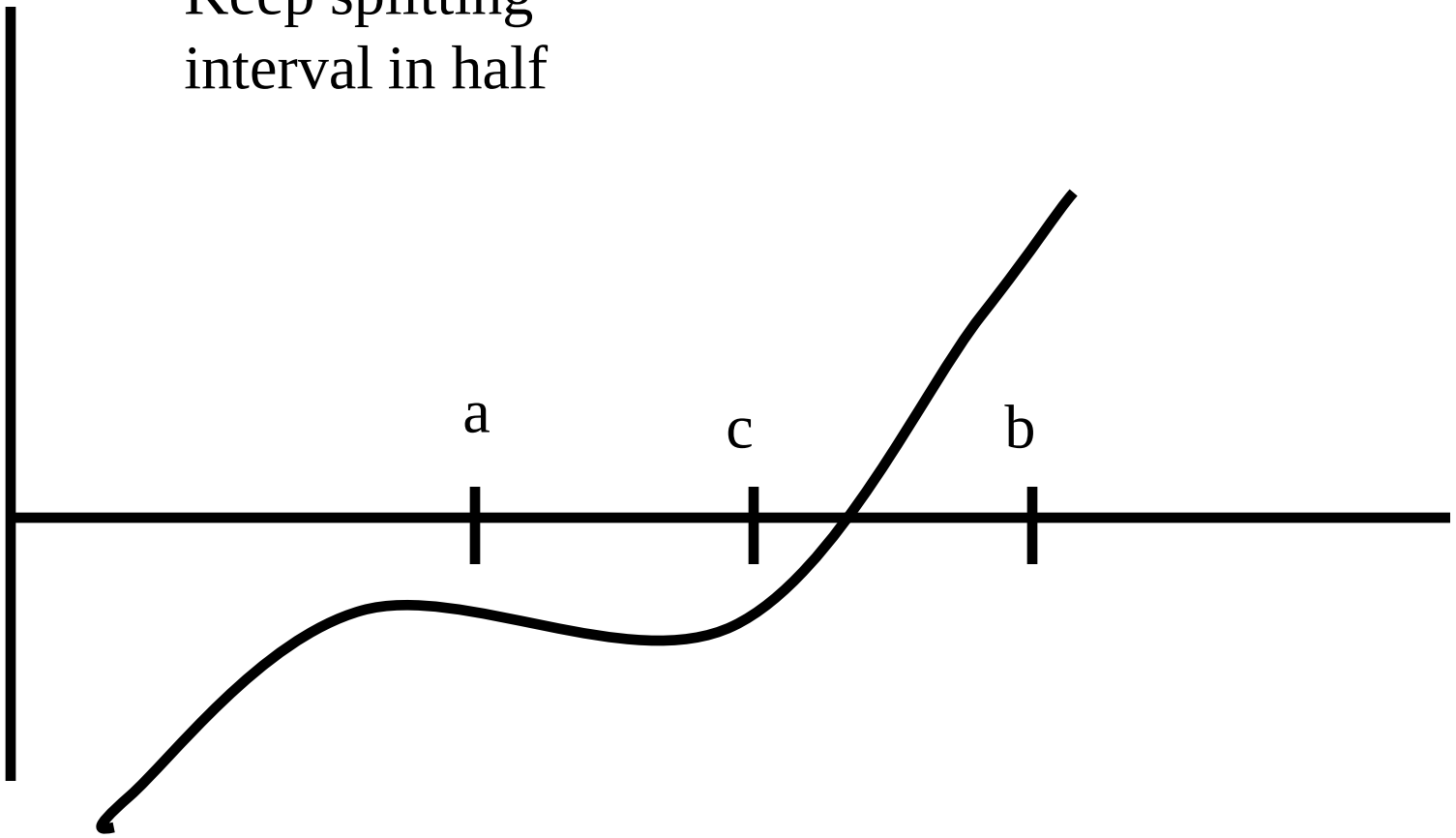

Bisection

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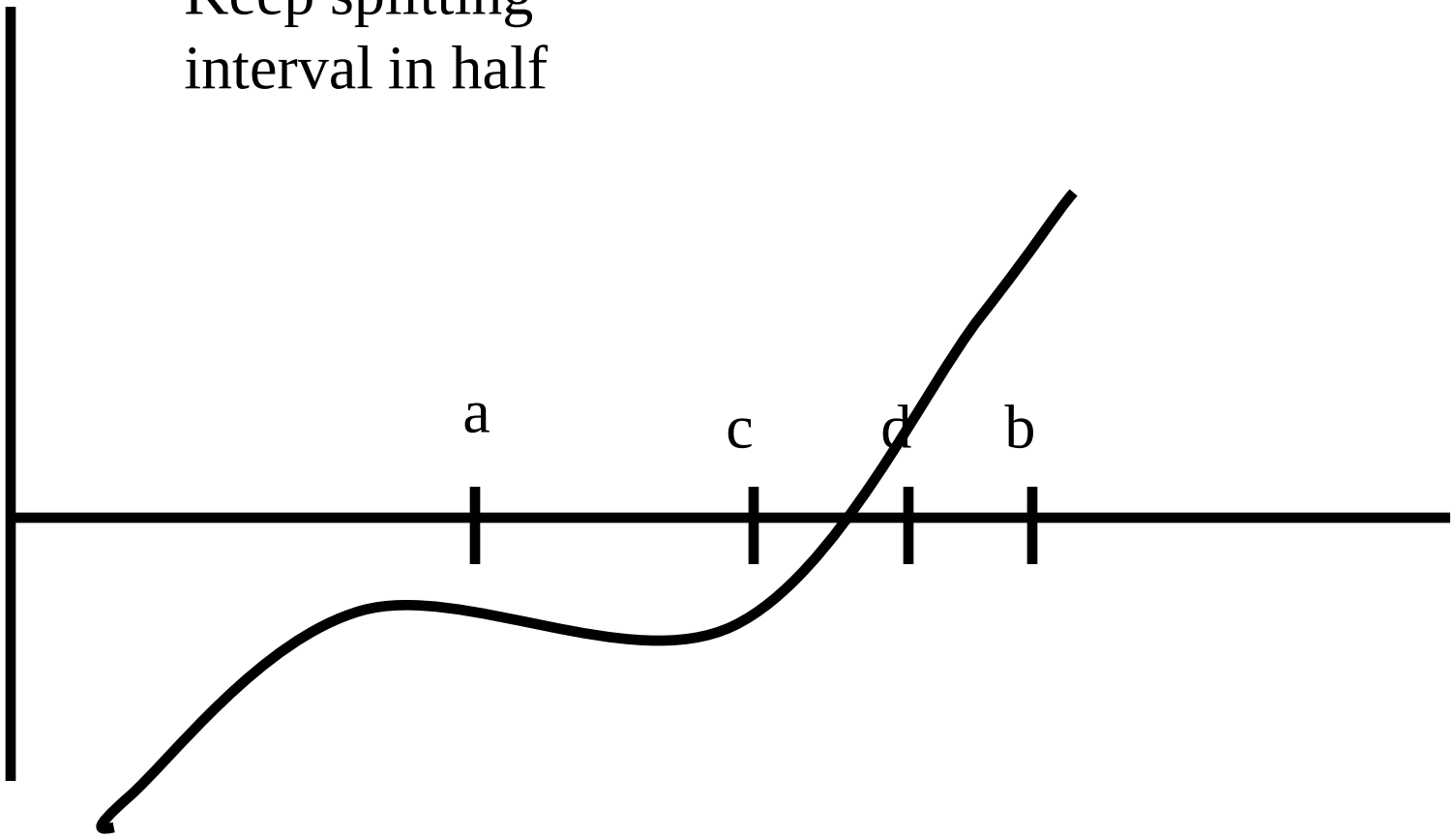

Bisection

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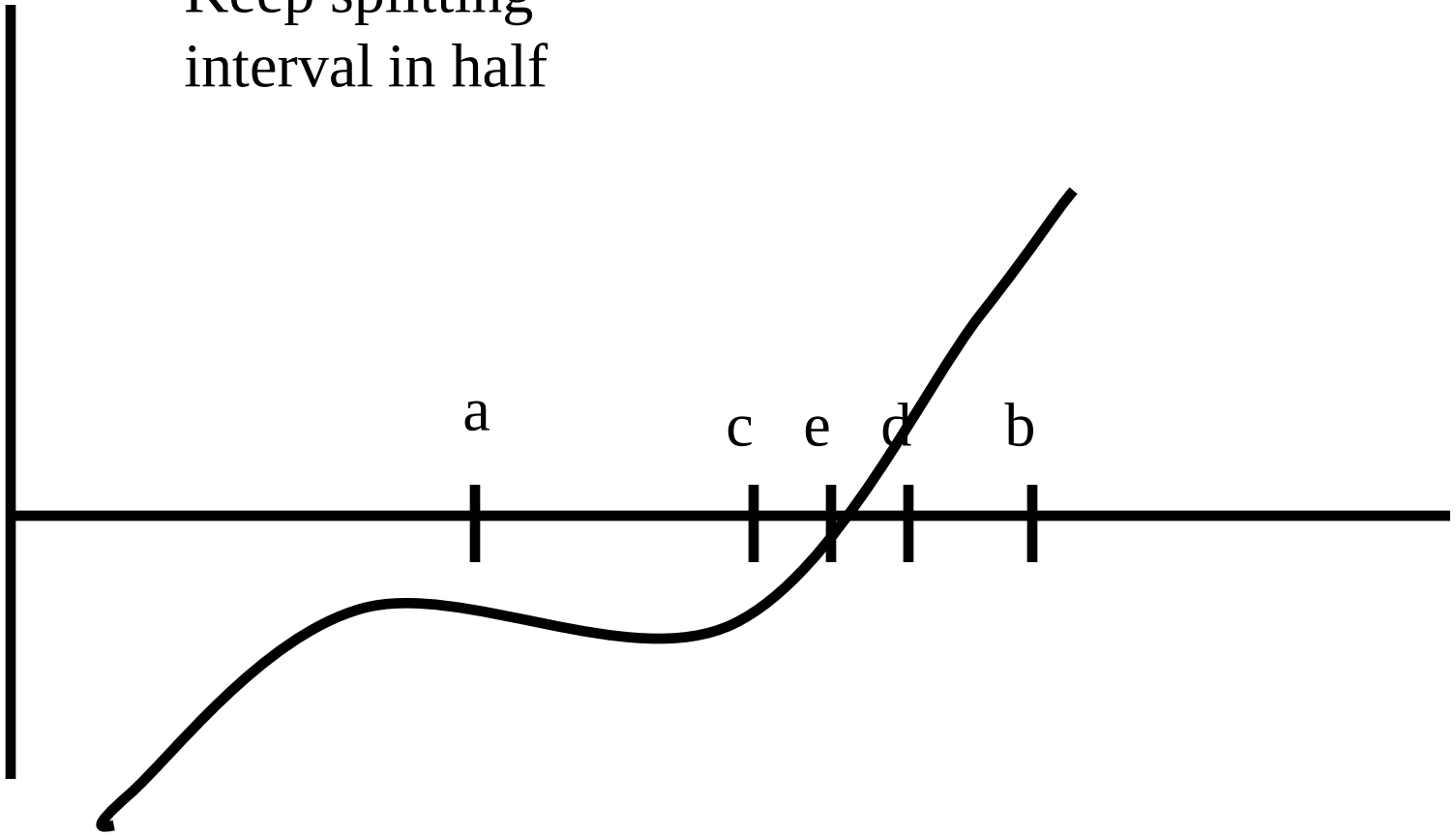
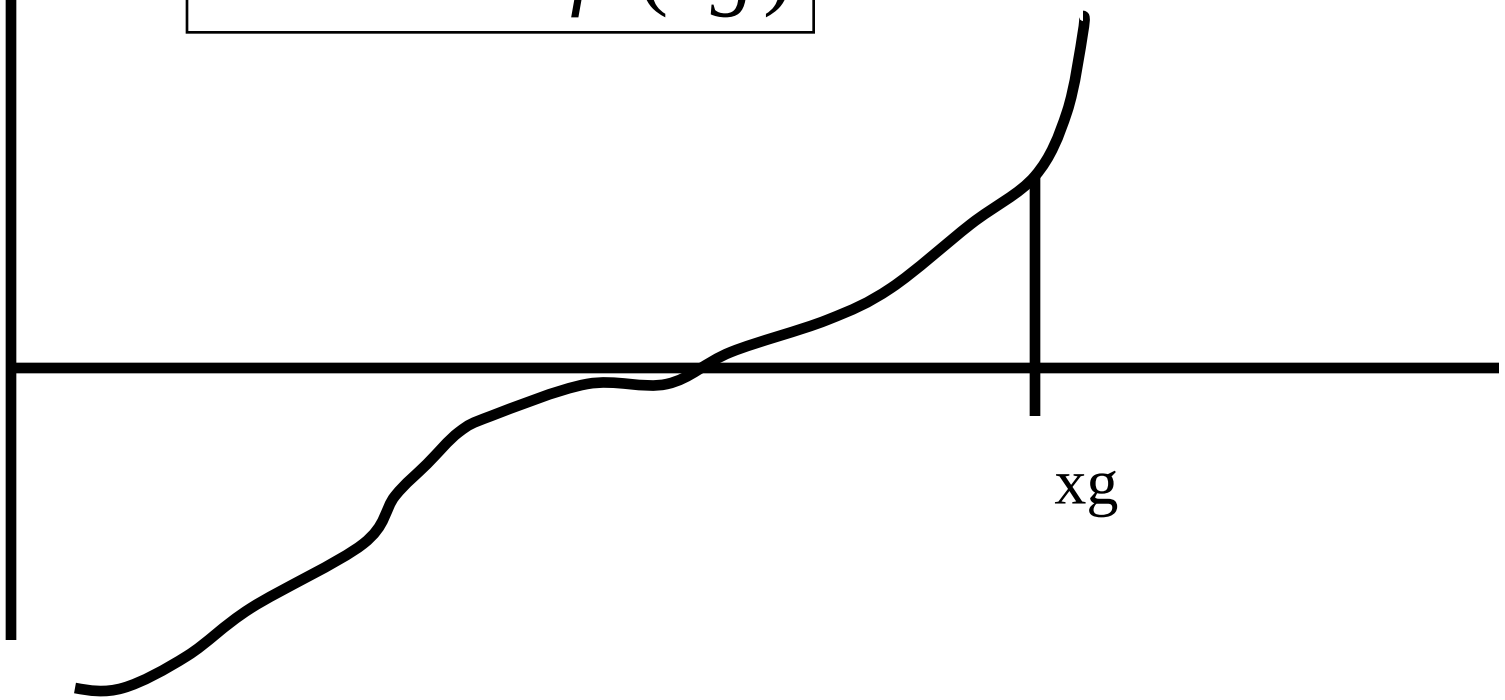


Newton's Method

$$x0 \approx xg - \frac{f(xg)}{f'(xg)}$$



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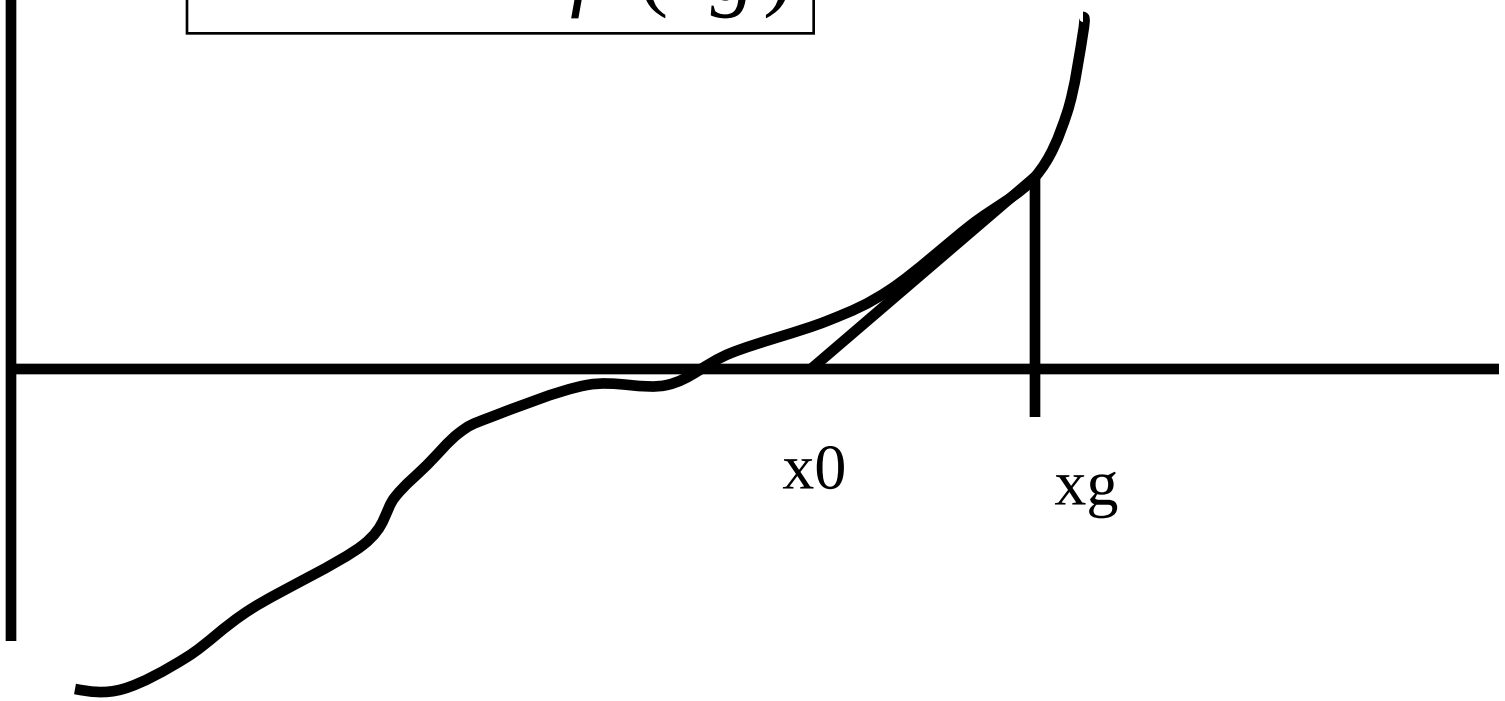

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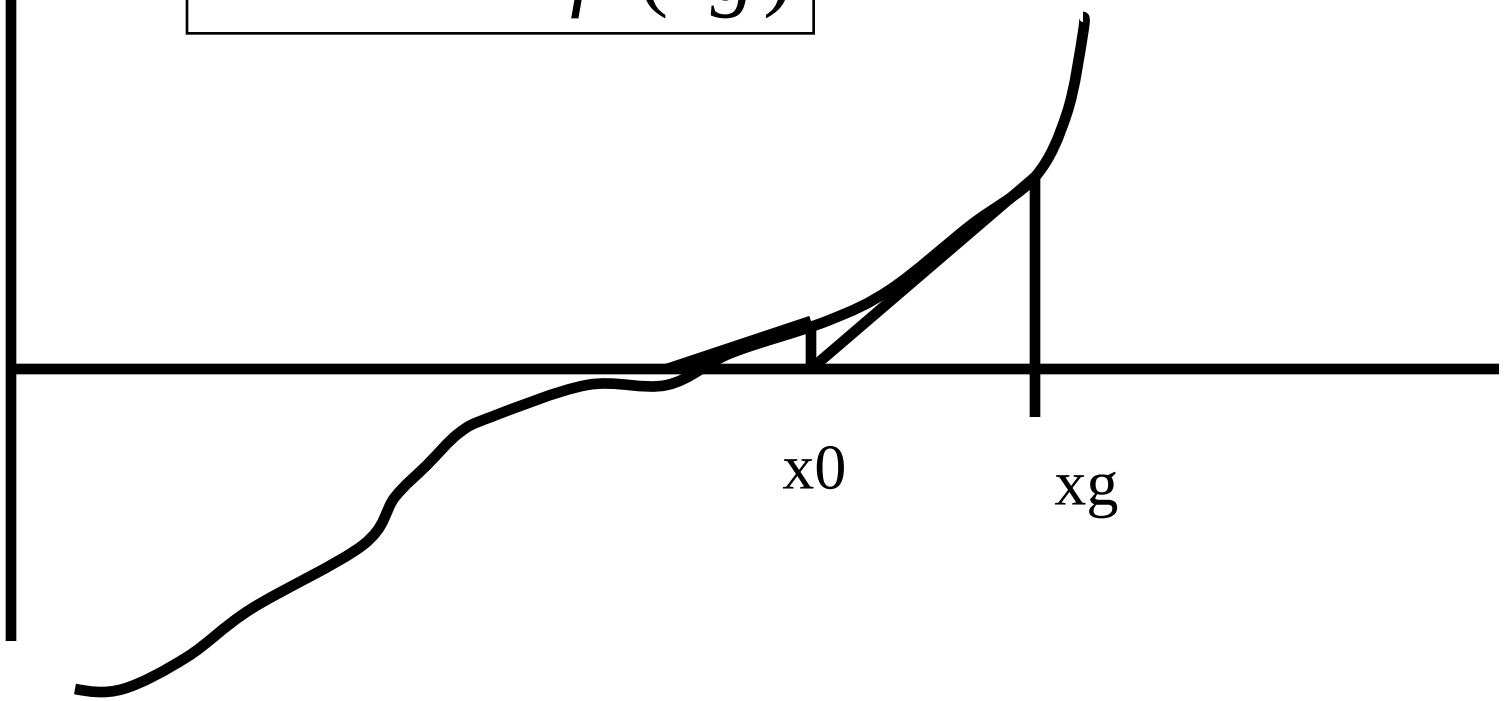
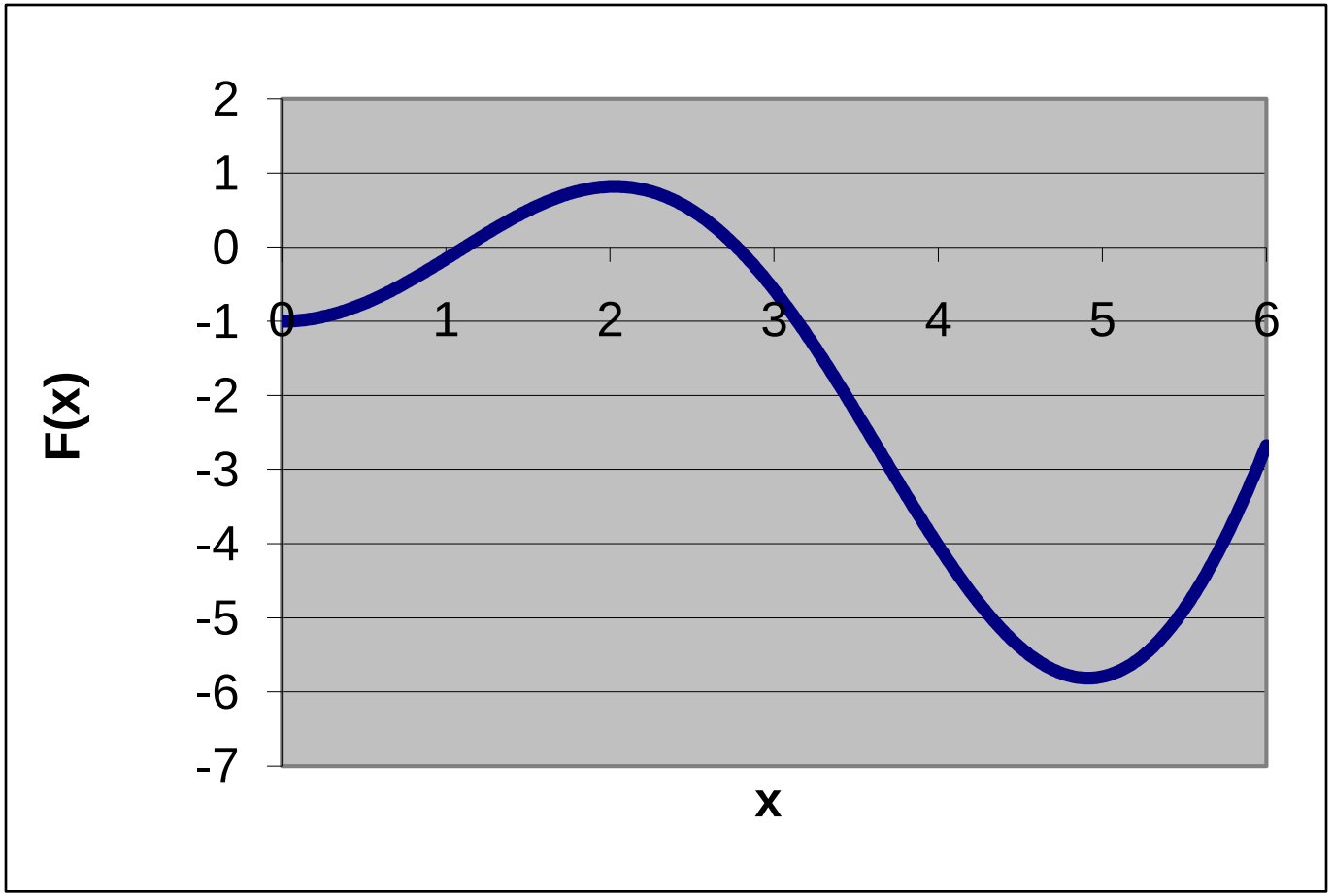


An Example

F(x)=x*sin(x)-1



Matlab

- **Use fzero function**

```
function findroot
guess=1
fzero('f',guess)
%
function func=f(x)
func=x*sin(x)-1;
```

Practice

- Find roots of x*sin(x)-1 on interval 0<x<12
  - First plot and estimate roots
  - Then run fzero to get more accuracy

Practice

- The temperature of the ground at a depth x for surface temperature T_s and initial temperature T_i is given on the next slide
- How deep should a water main be buried if we want to keep the water from freezing if the surface is at -15 C for 60 days?

Parameters

- t=60*24*3600
- T_s=-15 C
- T_i=20 C
- T=0 C
- α=1.38*10^{-7} m^2/s

$$\frac{T - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Practice

$$1+\cosh(\beta L)\cos(\beta L)=0$$

- Find first two positive values of β that solve this equation for L=4.2 m
- For EI=21,000 N-m^2 and rho=0.53 kg/m, calculate the frequencies from

$$\omega=\beta^2\sqrt{\frac{EI}{\rho}}$$

Questions?