Interpolation

Jake Blanchard
University of Wisconsin - Madison
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Case Study

- You've taken data for measured temperature as a function of time from a hot water faucet

Data

Time (s)	Temperature (F)
0	72.5
1	78.1
2	86.4
3	92.3
4	110.6
5	111.5
6	109.3
7	110.2
8	110.5
9	109.9
10	110.2

Questions

- Estimate temperature at t=0.6, 2.5, 4.7, and 8.9 seconds
- Estimate time it will take to reach T=75, 85, 90, and 105 degrees

Interpolation

- Defining a function that takes on specified values at specified points
- Unlike curve fits, interpolation always goes through the data points
- Generally piece-wise, rather than covering entire range
- Often, first approach is to draw straight lines between points

Polynomials

- For N data points, there is a unique polynomial (usually of order n-1) that goes through each point
- This is an interpolating polynomial, because it goes exactly through each data point
- Problem: between data points, function can vary by large amount

Piecewise linear interpolation

- Connect each data point by a straight line

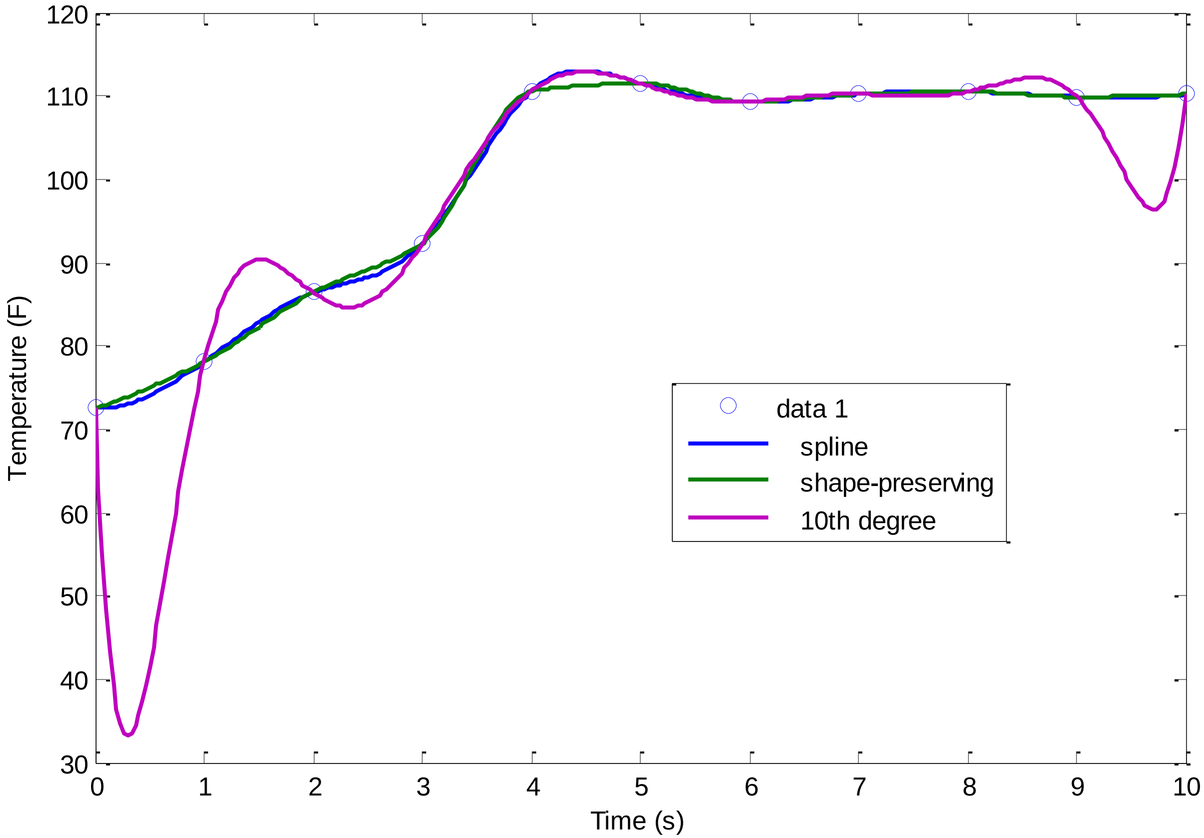
Piecewise Cubic Interpolation

- Goes through data points and has continuous first and second derivative from piece to piece

Shape-Preserving Interpolation

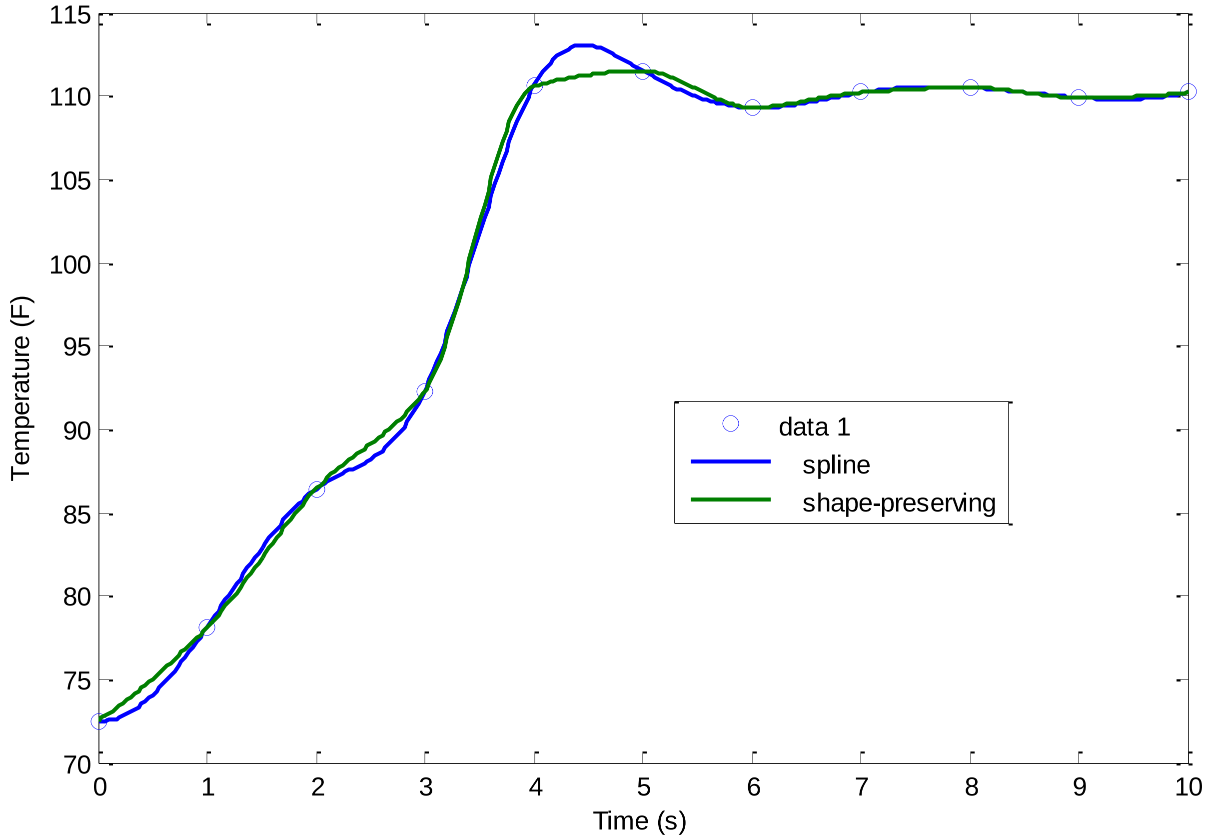
- Can be thought of as “visibly appealing”
- Abandon continuous second derivative
- Try to avoid large deviation between points

Example



120
110
100
90
80
70
60
50
40
30
Temperature (F)
0
1
2
3
4
5
6
7
8
9
10
Time (s)
data 1
spline
shape-preserving
10th degree

Without Polynomial



Matlab functions

- interp1 – 1-D linear interpolation
- interp2 – 2-D linear interpolation

1-D interpolation

- **yi = interp1(x,y,xi,'linear')**
- **yi = interp1(x,y,xi,'cubic')** – shape-preserving
- **yi = interp1(x,y,xi,'spline')**
- x,y=data vectors
- xi is vector of interpolation points

Script

```
time=0:10;
temps=[72.5 78.1 86.4 92.3 110.6 111.5 109.3
   110.2 110.5 109.9 110.2];
plot(time,temps,'o')
xlabel('Time (s)')
ylabel('Temperature (F)')
plotvals=0:0.1:10;
yvals=interp1(time,temps,plotvals,'linear')
hold on
plot(plotvals,yvals)
yvals=interp1(time,temps,plotvals,'cubic')
plot(plotvals,yvals,'r')
yvals=interp1(time,temps,plotvals,'spline')
plot(plotvals,yvals,'g')
```

Practice

- Computer controlled machines are used to shape a car fender
- The points on the next slide define the fender
- Use interpolation to define the entire fender

Fender Data

X (ft)	0	.25	.75	1.25	1.5	1.75	1.875	2	2.125	2.25
Y	1.2	1.18	1.1	1	0.92	0.8	0.7	0.55	0.35	0

Practice – Trace of My Hand

- Download and run **handdata.m**
- Plot x vs. y
- Let t=1:76
- Interpolate x vs. t and y vs. t
- Now plot curve for hand vs. data

Questions?