Homework 4

Problem 1

- A modified fox/rabbit problem is provided below. Solve it to t=15. Assume R=400 and α=0.01.

$$\frac{dr}{dt} = 2\left(1 - \frac{r}{R}\right)r - \alpha rf$$

$$\frac{df}{dt} = -f + \alpha rf$$

$$r(0) = 300$$

$$f(0) = 150$$

Problem 2

- The Lorenz equations represent a vastly simplifed weather model. Solve them for 0<t<30. Take β=8/3, σ=10, and ρ=28.

$$\frac{dx}{dt} = -\beta x + yz$$
$$\frac{dy}{dt} = -\sigma(y - z)$$
$$\frac{dz}{dt} = -yx + \rho y - z$$

$$x(0) = 27$$
$$y(0) = 9$$
$$z(0) = 12$$

Problem 3

- Solve this again for ρ=5.

$$\boxed{\begin{aligned} x(0) &= 4 \\ y(0) &= 3 \\ z(0) &= 4 \end{aligned}}$$

Problem 4

- Solve the boundary value problem shown below. Compare to $y=\sin^2(x)$

$$\frac{d^2y}{dx^2} = 2 - \frac{4y^2}{\sin^2 x}$$

$$\frac{dy}{dx}(1) = 2\sin(1)\cos(1)$$

$$y(2) = \sin^2(2)$$